Let's Solve Water

Technical Note
SonTek, a Xylem brand • XA00028

Range, Velocity, Sound Speed, and Snell's Law

HOW SOUND SPEED AFFECTS RANGE ACCURACY VS VELOCITY ACCURACY

As acoustic Doppler profiling systems are deployed for more diverse applications around the world, users will need to understand how the speed of sound affects the accuracy of acoustic measurements.

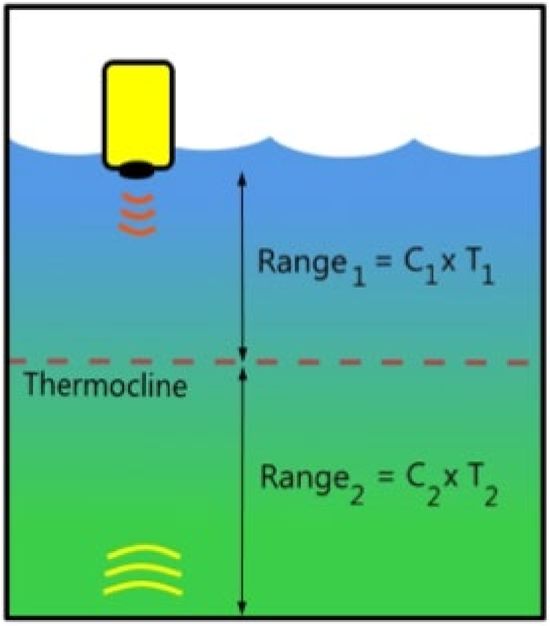
For this discussion, we will focus specifically on the difference between range accuracy and velocity accuracy. This is often a source of confusion for novice users because range accuracy is highly dependent on the sound speed profile, while velocity accuracy is not. Let's take a look at how sound speed affects both types of measurements.

With acoustics, we don't actually "measure" length or distance like we would with a ruler. Instead we transmit an acoustic ping into the water and record the time it takes for the echo to return from the bottom. We convert this elapsed travel time to a distance by multiplication.

The accuracy of this conversion depends on how much we know about the sound speed throughout the entire water column. This "profile" of the sound speed will tell us how fast the acoustic ping is moving as it travels through layers of different temperatures, salinities, or densities. We can apply the following equation to calculate range from time using the sound speed profile.

$$Z = \int_0^{T/2} C(z)\cos\theta \, dt$$

Z is the calculated range in meters, *C* is the speed of sound, *T* is the time interval between transmit and receive and θ is the angle of the beam from vertical. We divide the time in half because the ping travels the distance twice, once to the bottom and then back again. Because acoustic Doppler profiling systems can measure time intervals very accurately, the range calculation accuracy ultimately depends on how well the speed of sound is known. The major point is that we need to know the speed of sound not only at the transducer, but *over the entire water column* to accurately calculate range. Range errors can be very troublesome near inlets and river mouths where the temperature, salinity, and density of the water can change rapidly over time and space.



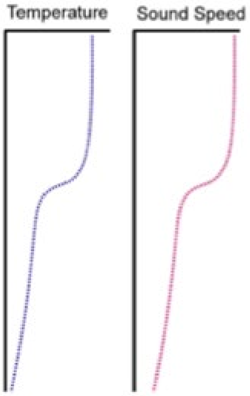


Figure 1: The figure above illustrates a simple range calculation using a sound speed profile with two distinct layers. C is the speed of sound and T is the elapsed ping travel time in each layer. To measure the total range to the bottom accurately, we would have to directly measure the speed of sound profile with a profiling instrument like a CTD. The total range is then the sum or integration of the sound speed over time in each layer.

a xylem brand

So, now that we know how to make accurate range measurements, how do we make accurate velocity measurements? And, how does the sound speed influence these velocity measurements? Interestingly, an accurate velocity measurement only depends on knowing the sound speed at the transducer. So it is actually easier to measure velocity than it is to measure range. This may seem a bit unusual after the previous discussion, but range and velocity are very different kinds of measurements.

To measure velocity, we leave the time domain behind and focus on the change in frequency (or Doppler Shift) of the acoustic signal. Because we are now concerned with frequency and not time or distance, we don't need to know the entire sound speed profile. We just need to know what it is at the transducer. The reason for this is Snell's law.

Snell's Law is a fairly simple equation that can be used to calculate the change in the speed and the direction of a wave when it travels from a medium of one density into a medium of another. This is also called refraction. No matter what the density change, the ratio of the speed and direction of a wave remains constant. Because an acoustic ping is a wave (of sound); it too must obey Snell's Law.

$$\frac{\sin\theta_1}{C_1} = \frac{\sin\theta_2}{C_2}$$

0_1 and 0_2 are the angles of the acoustic beam in two adjacent density layers and C_1 and C_2 are the sound speeds in each layer. If there are multiple layers with different sound speeds, we would apply this equation between each layer.

As the acoustic wave travels into an area of colder denser water it is refracted or bent away from its original transmit angle. Note that the angles in this figure are exaggerated. The actual change in the beam angle depends on how large the change in density is from one layer to the next. The main point to remember; is that any acoustic energy reflected back to the transducer has traveled the same path twice, from the transducer to the bottom and back again.

Continuing with the example illustrated above, let's take a look at where and how the sound speed is applied in the velocity calculation. We begin by measuring the Doppler shift in each layer as D_1 and D_2.

$$D_1 = \frac{-2U_1 F\sin\theta_1}{C_2} \text{ and } D_2 = \frac{-2U_2 F\sin\theta_2}{C_2}$$

$$\frac{\sin\theta_1}{C_1} = \frac{\sin\theta_2}{C_2}$$

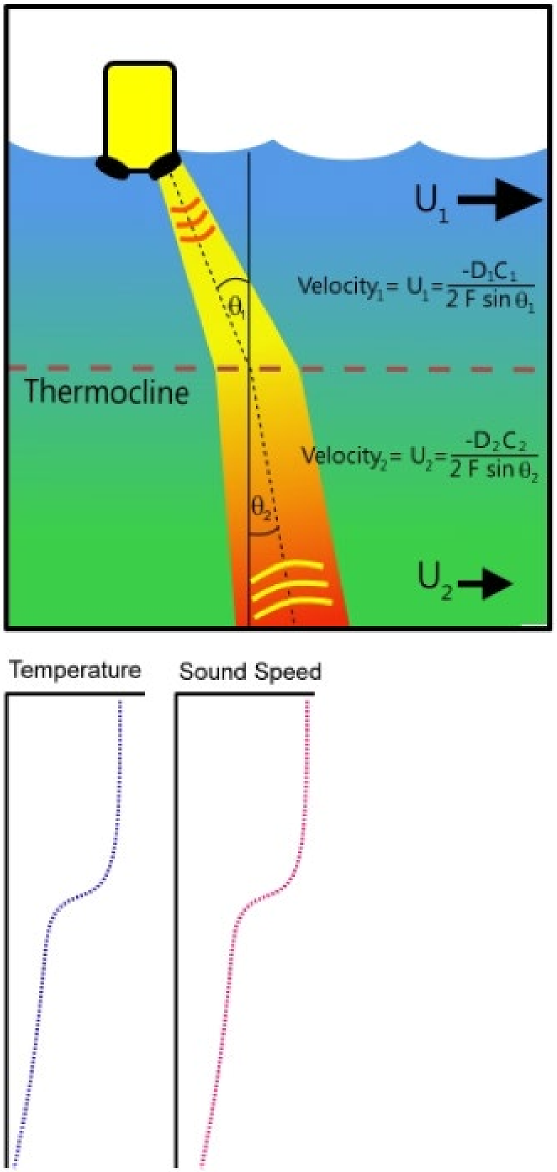


Figure 2: The figure above illustrates another two layer scenario where the path of an oblique acoustic beam is altered by a Thermocline. A Thermocline is defined as the location of an abrupt change, or gradient, in temperature when moving down or up through the water column. This change in temperature will also cause a change in the local speed of sound.

Applying Snell's Law by substitution into the second equation we get

$$D_2 = \frac{-2U_2\, F\sin\theta_1}{C_1}$$

We can rearrange this equation to calculate velocity.

$$U_2 = \frac{-D_2\, C_1}{2F\sin\theta_1}$$

Now the velocity in layer 2 can be calculated using the speed of sound from layer 1. By using Snell's Law, we are able to calculate the water velocity in layer 2 from the measured Doppler shift in layer 2 and the sound speed from layer 1 *at the transducer*. The key is that the change in the angle of the beam always balances the change in sound speed.

Measuring accurate water velocities anywhere in the water column is purely a function of the frequency of the transducer F, the angle of the transducer 0, and the speed of sound C at the transducer. Remember, this only works because of Snell's law and because we are measuring the change in frequency (the Doppler Shift). Consequently, the accuracy of velocity measurements relies only on knowing the speed of sound at the transducer and not over the whole profile. Thank you Mr. Snell!

So, before you invest in a profiling CTD, you should decide if it is absolutely necessary for the data you are collecting. Depending on your acoustic Doppler profiling system application, you may or may not need to measure the sound speed profile. If you are measuring water velocities, currents profiles or discharge, the sound speed profile isn't necessary for accurate velocity measurements. However, if you are concerned with the precise vertical location of the velocity cells in the water column, a sound speed profile will help improve this calculation. But, accurate velocity measurements only need the sound speed at the transducer because of Snell's law.

If you are measuring acoustic range during a bathymetric survey with an acoustic Doppler profiling system or any other transducer, you should definitely measure the sound speed profile. This will greatly improve the accuracy of your range measurements especially in areas where the water temperature, salinity, and density are inconsistent. Should you require a profiling CTD for your acoustic Doppler profiling system application, SonTek has several options available to fit your needs.

SonTek, a Xylem brand
9940 Summers Ridge Rd.
San Diego, CA 92121

☎ +1.858.546.8327
✉ inquiry@sontek.com
🌐 SonTek.com

Sound Principles.
Good Advice.

SonTek.com